Journal of Material Science and Mechanical Engineering (JMSME)
Print ISSN: 2393-9095; Online ISSN: 2393-9109; Volume 2, Number 3; April-June, 2015 pp. 270-274

http://www.krishisanskriti.org/jmsme.html

Kinematics and Workspace analysis of a 6-DoF SPS Parallel Manipulator

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Abstract—*This paper addresses the evaluation of total workspace and kinematics of selected configuration of 6-DoF fully parallel manipulator. Starting from the inverse nominal kinematic model of parallel manipulator, a Matlab code was written to solve the inverse kinematic model for workspace evaluation. Usable workspace has been also found out which is the subset of total evaluated workspace and defined by the application of the manipulator system. Furthermore a comparison between theoretical and actual workspace has been made. Actual workspace has been calculated experimentally. The comparison shows that actual workspace is less than theoretical workspace.*

Keywords: *Parallel Manipulator, Workspace, Kinematics.*

1. INTRODUCTION

A Parallel manipulator is very useful and highly accurate six-axis robotic platform. It has attracted attention of many researchers and mathematicians due to its wide range of applications in nuclear, medical, cryogenics and other emerging fields where minimal human involvement, greater automation and better accuracy is required. These mechanisms have closed loop kinematics. Many challenges however, exist in this field as of now from the point of view of configuration, design, and control to the improvement of its accuracy. Lot of research is going on in the aspect of design, configuration and its control. This paper addresses the issue of workspace and kinematics of 6- axis parallel manipulator. Workspace analysis and optimization are important in a manipulator design. As a parallel manipulator (PM) has a much smaller workspace than that of its serial counterpart, the workspace quantity and quality have become the most important performance indices **[1–3]**. The solution of this problem is critical in the design and motion planning of the manipulator. Therefore, geometric parameters of the links, joints, base and moving platform should be designed to optimize the workspace. Workspace is defined as the space reachable by the moving platform of 6-DoF parallel manipulator machine. Workspace shape and its volume are determined by the kind of mechanical architecture and the angular and linear range of displacement of each actuator. Workspace of the parallel manipulator also depends on maximum and minimum length of the actuators. Workspace determination is an important step in the design and kinematic modeling of any parallel manipulator architecture. It plays an important role in the time of singular points determination of the parallel manipulator also. **[4] [5]**. The determination of workspace of a general parallel manipulator is still challenging, due to the following reasons:

(1) As the complete workspace of a 6-DOF manipulator is embedded into a 6-D space, it cannot be represented it graphically in a human readable way. So our concern here only considered the 3-D workspaces which are subspaces of the complete 6-D workspace.

(2) There is no general way to analytically determine the 6-D workspace boundary and volume for a 6-DOF PM, especially when the joint limits and mechanical interference are considered. Therefore, the 3-D reachable position workspace volume is employed here in this paper.

The workspace boundaries are obtained by the intersection of the reachable workspace generated by each actuators.

2. NOMINAL MODEL OF PARALLEL MANIPULATOR

A typical 6-axis parallel manipulator consists of a moving platform that is connected to a stationary base through six numbers of parallel linear independent actuators with the help of spherical and /or universal joints at the ends **[6]** as shown in Fig. 1 and Fig. 2. This machine has been developed in centre for design and manufacturing, BARC (Mumbai), India.

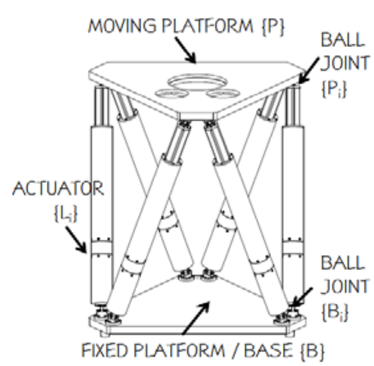


Fig. 1: Six-axis parallel manipulator

This machine is capable to move in three linear directions and as well as in three angular directions in 3-D space. A very promising application of this six-axis machine is in the positioning and orientation of instrumentation in synchrotron radiation beam lines in RRCAT (Indore), India. Some of other applications are Tool Control for Precision-machining & Manufacturing, Positioning of Optics.

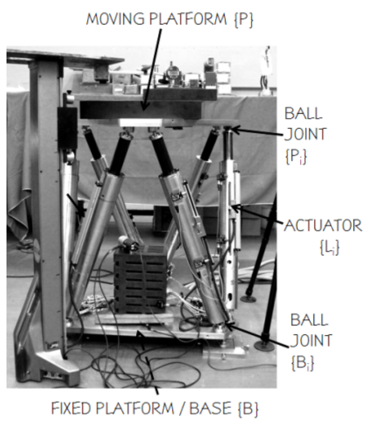


Fig. 2: Six-axis parallel manipulator developed at CDM, BARC

The nominal kinematic model of the parallel manipulator is an ideal model which is based on following assumptions; 1) All the ball joints can be treated as points and the links as straight lines of known length which are connected between the joint centers. 2) The actuators are perfectly assembled to the joints so that each actuator axis passes through the respective joint centers. 3) The platform structure is rigid and the locations of the joints are precisely known. 4) There is no deformation in the actuators and other components **[7] [8].**

The joint pairs attached to the moving plate and the base plates are denoted by P_1 to P_6 and B_1 to B_6 respectively. It is assumed that the spherical joints on the base and moving plate form a circle as shown in Fig. 3. Refer to Fig. 3 and Fig. 4, coordinate frames {B} and {P} are arbitrary embedded in the centre of base and centre of the platform with vectors referenced in coordinate frame {B} denoted by B_v and vectors referenced in {P} denoted by P_v.

Z-axis is perpendicular to the base plane. The joints on the base are denoted by B_i and the position of the joint centers with respect to (B} are given by the vectors ${}^{B}b_i$. Similarly P_i denotes movable platform joints with the location of the joint centers given by ${}^{P}p_i$ (or ${}^{B}u_i$ in the base frame). The six legs are denoted by L_i with a vector from B_i to P_i defined by $\lambda_i\ {}^{B}l_i$. Where λ_i is the magnitude (the length of the leg) and ${}^{B}l_i$ is a directional unit vector. Each leg contains a length monitoring device as a linear scale so λ_i that is a measurable quantity. A vector Λ containing all λ_i will be used to describe the measured leg lengths through a displacement vector ${}^{B}q$ and through a rotation vector ${}^{B}R_{p}$ **[6]**.

The vector ${}^{B}q = [P_x\ P_y\ P_z]^{T}$ gives the relative displacement of the origin of {P} from the origin of {B}. Where P_x, P_y, P_z are the translation motion of moving platform in x, y and z direction respectively. Rotation matrix ${}^{B}R_{P}$ is formed using the roll-pitch-yaw angles α, β and γ respectively. So finally an orientation vector can be defined by $\Omega = [\alpha\ \beta\ \gamma]^{T}$ where C, S stands for cosine and sine respectively. Rotation matrix ${}^{B}R_{P}$ is obtained from the three Euler angles α, β and γ **[6]**. The inverse kinematics deals with calculating the leg lengths when the position and orientation of moving platform is given.

$$\Omega = {}^{B}R_{P} = Rot(z,\gamma).Rot(y,\beta).Rot(x,\alpha) =$$

$$\begin{bmatrix} C\gamma\, C\beta & C\gamma\, S\beta\, S\alpha - S\gamma\, C\alpha & C\gamma\, S\beta\, C\alpha + S\gamma S\alpha \\ S\gamma\, C\beta & S\gamma\, S\beta\, S\alpha + C\gamma\, C\alpha & S\gamma\, S\beta\, C\alpha - C\gamma S\alpha \\ -S\beta & C\beta\, S\alpha & C\beta\, C\alpha \end{bmatrix}$$

In effect, it maps global pose to local actuator lengths i.e. Π to Λ. Where $\Pi = [P_x, P_y, P_z, \alpha, \beta \text{ and } \gamma]$ is the pose of the moving platform. With the assumptions of the nominal model, the inverse kinematics are uncomplicated and yielding a closed form solution. After noticing that the platform joint location vectors are related by ${}^{B}u_i = {}^{B}R_{p}\ {}^{P}p_i$, the vector chain in Fig. 4 can be written as

$$\lambda_i\ {}^{B}l_i = {}^{B}u_i + {}^{B}q - {}^{B}b_i \quad (1)$$

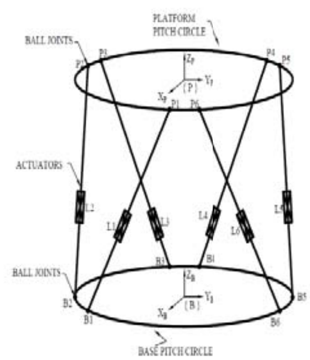


Fig. 3: Nominal Model of the 6-axis
Parallel manipulator

The nominal inverse kinematics are then solved by taking the magnitude of equation (1)

$$\lambda_i = |\lambda_i\ {}^{B}l_i| = |{}^{B}u_i + {}^{B}q - {}^{B}b_i| \quad (2)$$

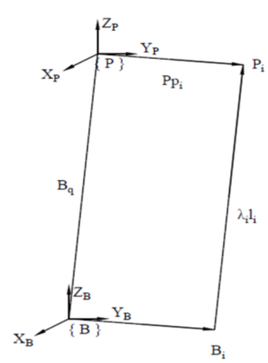


Fig. 4: Nominal Vector Chain for i^th Leg

3. GEOMETRICAL PARAMETERS AND WORK SPACE CALCULATION OF SELECTED 6-DOF PARALLEL MANIPULATOR CONFIGURATION

Workspace is depends upon geometrical and dimensional characteristics of the machine. If there are variations in these parameters, then obviously workspace will change for that machine. So it is very necessary to define all the dimensional and geometrical parameters of a machine for the determination of the workspace. Table 3.1 shows the dimensional parameters of the selected configuration of 6-SPS parallel manipulator in tabular form.

Table 1: Geometrical parameters of selected configuration

Dimensional Parameters	Value
Radius of fixed base (R in mm)	469.985
Radius of platform (Rp in mm)	470.009
Spherical joint distribution angle of the Fixed platform α b (in degree)	45
Spherical joint distribution angle of the moving Platform α p (in degree)	45
Minimum length of the actuator (L min. in mm)	865
Maximum length of the actuator (L max in mm)	1175
The distance of joint between B1 and B2 (b, mm)	800
The distance of joint between B1 and B6 (d, mm)	160
Height of the manipulator (H, mm)	1150

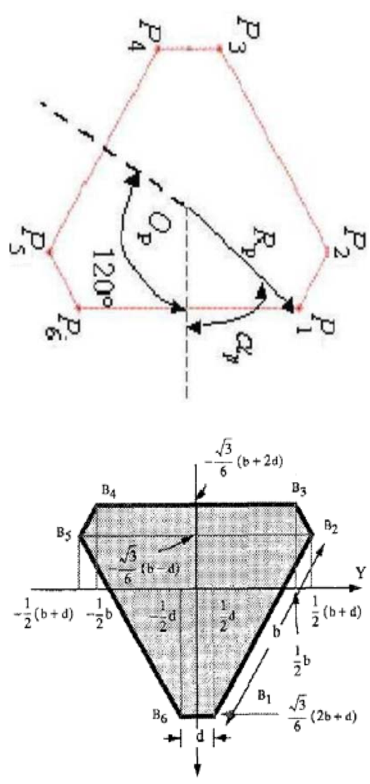


Fig. 5: Dimensional parametres of (a) Moving plate (b) Fixed plate

Fig. 5 (a) & (b) shows the dimensional parameters of the moving platform and base platform in graphical form which will help to determine the workspace of selected manipulator system. A significant characteristic of Parallel manipulator is that the workspace is not constant, but varies with platform orientation.

A MATLAB code was written to solve the inverse kinematics for workspace evaluation for this selected 6-SPS parallel manipulator. MATLAB which consists of dimensional parameters of both the platform and the minimum and maximum length of the linear actuators as given in table 1. Some of the obtained results are presented with the help of Fig. 6 and Fig. 7. Fig. 6 shows when the platform is at $P_x = 0$, $P_y = 0$, $P_z = 0$ (Translational movement of platform) and free to rotate about all three axis. Fig. 7 shows the workspace of manipulator when the platform is only translated in all three axis means α, β and γ = 0 (Rotational movement) is equal to zero.

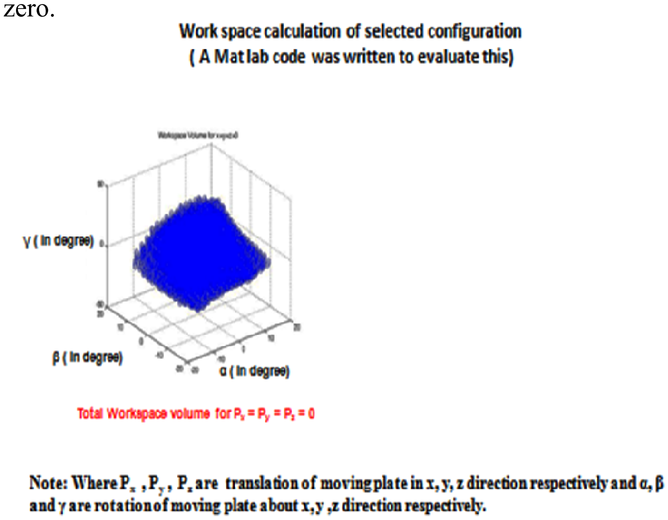


Fig. 6: Workspace volume for $P_x = P_y = P_z = 0$

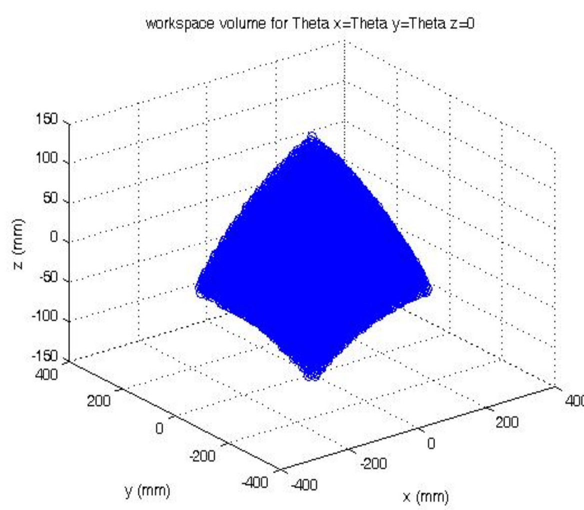


Fig. 7: Workspace volume of for α, β and γ = 0

Table 2 and Table 3 show the comparison between theoretical and actual results of workspace. Theoretical results of workspace can be obtained from the code written in MATLAB. The actual results have been found out from experiment.

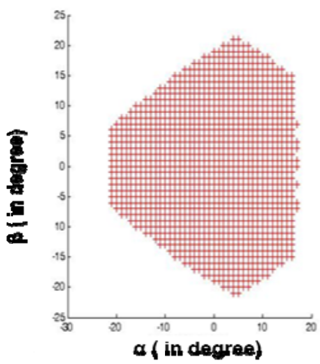
Table 2: Rotational Range of parallel manipulator

	Theoretical	Actual
α	± 20 degree	± 19 degree
β	± 20 degree	± 19 degree
γ	± 40 degree	± 38 degree

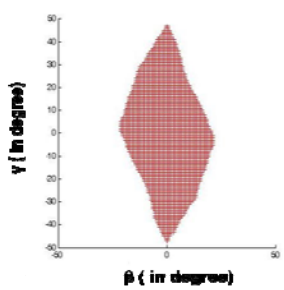
Table 3 Translational Range of parallel manipulator

	Theoretical	Actual
Px	± 130 mm	± 125 mm
Py	± 130 mm	± 125 mm
Pz	± 120 mm	± 115 mm

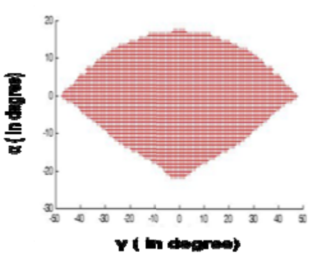
Fig. 8 to Fig. 9 shows the workspace of the parallel manipulator machine in graphical manner for different conditions.



(a)

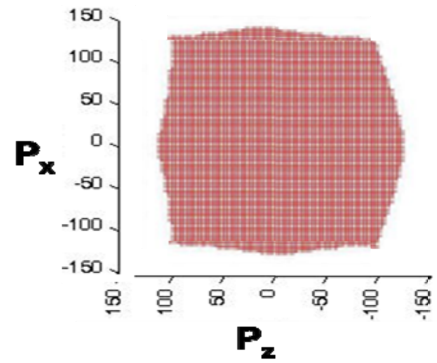


(b)

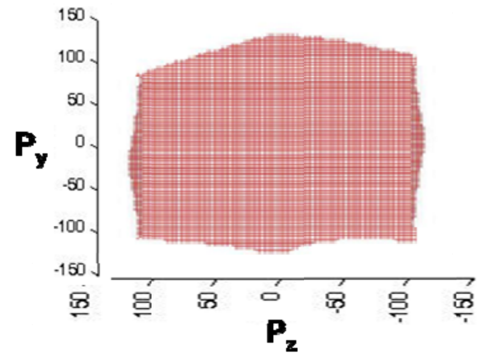


(c)

Fig. 8: (a) workspace as seen in x-y plane (b) workspace as seen in y-z plane (c) workspace as seen in z-x plane



(a)

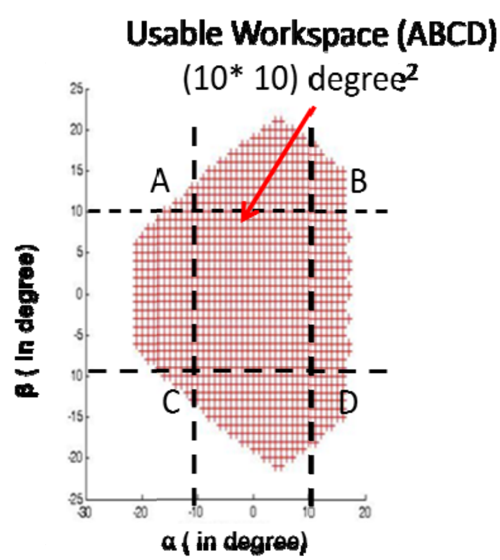


(b)

Fig. 9: (a) workspace as seen in x-z plane (b) workspace as seen in y-z plane

4. USABLE WORKSPACE

Usable workspace is the subset of the total workspace. Usable workspace depends on application of the system. In this paper the translational usable workspace is ± 100 mm along all three axis's while the rotational usable workspace is ± 10 degree about all three directions has been taken. In Fig. 10 enclosed areas show the rotational usable workspace and in Fig. 11 enclosed areas show the translational workspace.



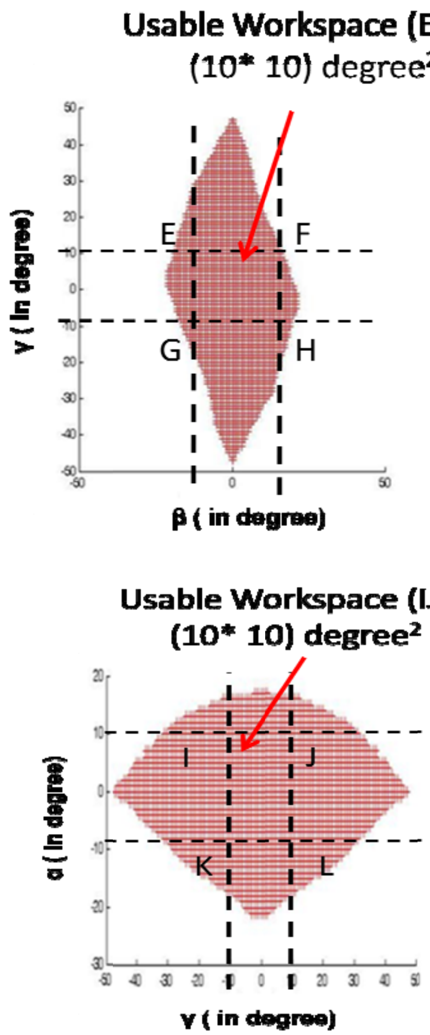
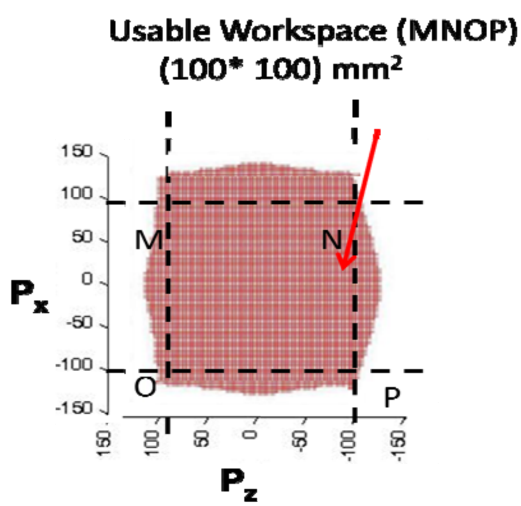


Fig. 10: Rotational usable workspace

(Enclosed areas)



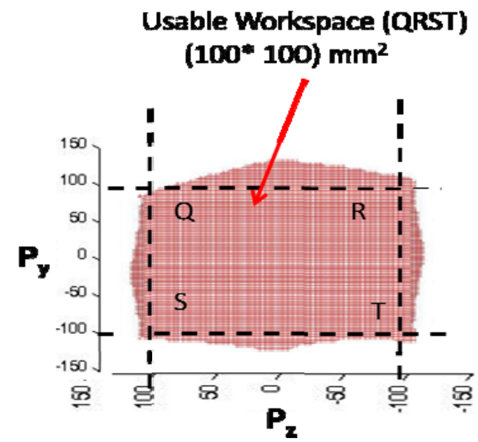


Fig. 11: Translational usable workspace
(Enclosed areas)

5. CONCLUSION

Nominal mathematical model of parallel manipulator has been discussed in this paper. In this paper, workspace for a given configuration has been calculated. A comparison between theoretical and actual workspace has been made. The comparison shows that actual workspace is less than theoretical workspace. The reasons of lesser actual workspace are limit switches and physical constraints. Usable workspace has been selected as per application of the system. Usable workspace is basically subset of total workspace.

6. ACKNOWLEDGEMENT

This project is supported by the Centre for Design and Manufacturing, BARC and Homi Bhabha National Institute, BARC, Mumbai (India).

REFERENCES

[1] **Kim D, Chung W, Youm Y**. "Geometrical *approach for the workspace of 6-dof parallel manipulators*". In: Proceedings of IEEE conference on robotics and automation, Albuquerque, New Mexico. p. 2986–91.

[2] **Monsarrat B, Gosselin C.** "*Workspace analysis and optimal design of a 3-leg 6-dof parallel platform mechanism*". IEEE Transactions on Robotics and Automation 2003; 19:954–66.

[3] **Badescu M, Mavroidis C.** "*Workspace optimization of 3-legged upu and ups parallel platforms with joint constraints*". ASME Journal of Mechanical Design 2004; 126:291–300.

[4] **Agrawal S. K.**, "*Workspace Boundaries of In-parallel manipulator Systems,*" International Journal of Robotics and Automation, Vol 7, No 2, 1992, pp 94-99

[5] **Agrawal S.K. and Roth B**., "*Statics of In- Parallel Manipulator Systems"*, Trans. ASME Journal of Mechanical Design, Vol 114, 1992, pp 564-568

[6] **Merlet J.P.**, "*Parallel Robots"*, 2nd edition, Springer Publications, 2006.

[7] **Lin and Ehmann,** "Error *Modeling and Compensation of Stewart Platform*", Annals of CIRP, vol.7/3, pp.395-406, 1993.

[8] **Veischegger, W. K., and Wu C.,** " *Robot Accuracy Analysis*", IEEE proceedings of international conference on cybernetics & Society, pp 425-430, 1985.